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LETTER TO THE EDITOR

A collective variable approach for dispersion-managed solitons

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Abstract

We present a method to express the generalized nonlinear Schrödinger equation, for pulse propagation in dispersion managed fibre-optic links, in terms of pulse parameters, called collective variables (CVs), such as pulse *width*, *amplitude*, *chirp* and *frequency*. The CV equations of motion are derived by imposing a set of constraints on the CVs, to minimize the soliton dressing during propagation.

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Many nonlinear physical systems can give rise to interesting *energy-localization* effects with a relatively long lifetime. In optics, a typical example of such an effect is called '*soliton*', i.e., a light pulse which can propagate in optical fibres over very long distances [1]. Recent studies demonstrated that the use of dispersion-managed (DM) solitons for data transmission purposes will substantially increase the capacity of the fibre-optic links [2–4]. Basically, dispersion-management techniques utilize a transmission line with a periodic dispersion map, such that each period is built up by two types of fibre, generally with different lengths and opposing group-velocity dispersion (GVD) [2–9].

The design and optimization of these soliton-transmission systems are fundamentally based on the general particle-like nature of solitons. This particle-like behaviour has led to the formulation of collective variable (CV) techniques for DM solitons, to gain more insight into their dynamical behaviour, since no exact analytical solution for DM solitons exists to date [3]. For such a soliton, it is therefore useful to incorporate many parameters in the theoretical treatment of its dynamics, called CVs, symbolically X_j $(j = 1, \ldots, N)$, associated with the pulse width, amplitude, chirp, frequency, and so on. To this end, one must decompose the original field, say $\psi(z, t)$ at position z in the fibre and at time t, in the following way

$$\psi(z, t) = f(X_1, X_2, \ldots, X_N, t) + q(z, t) \tag{1}$$

where the ansatz function f is chosen to be the best representation of the pulse configuration and $q(z, t)$ is the remaining field such that the sum of f and q satisfies the dynamical equation describing the soliton dynamics. The field q, that we call the *residual field*, accounts for the dressing of the soliton and any radiation coupled to the soliton's motion. In this Letter, we would like to point out a fundamental problem concerning the CV treatments for DM solitons that have appeared in the literature to date [2, 3, 8, 9].

All these CV treatments have the common feature that the *residual field* is completely ignored. The approximation of neglecting the *residual field*, called 'bare approximation' in condensed matter physics [10], can lead to dramatic consequences depending on the choice of the ansatz function [10]. In fact, *bare approximation* yields consistent results only when there is no considerable radiation and the dressing is also negligible. This insufficiency has already been noted in some recent studies, in which considerable effort was made to develop an accurate ansatz function for DM solitons (e.g., Hermite–Gaussian ansatz) [3, 8, 9]. This strategy of improving the ansatz for DM solitons is useful in the sense that it leads to a reduction in the dressing part of the residual field. Nevertheless, it will not account for any radiation of the residual field. In this Letter we present a rigorous CV treatment for the DM solitons which resolves this fundamental point by explicitly incorporating the residual field.

Nonlinear pulse propagation in fibre links may be described by the generalized nonlinear Schrödinger equation (NLSE) [1]:

$$\psi_z + \mathrm{i}\frac{\beta_2(z)}{2}\psi_{tt} - \mathrm{i}\gamma(z)|\psi|^2\psi = -\frac{\alpha(z)}{2}\psi + \frac{\beta_3(z)}{6}\psi_{ttt} - \mathrm{i}\gamma_{\mathrm{r}}(z)\,\psi(|\psi|^2)_t - \gamma_{\mathrm{s}}(z)(|\psi|^2\psi)_t \tag{2}$$

where $\psi(z,t)$ is the envelope amplitude of the electric field and the subscripts z and t denote the spatial and temporal partial derivatives. $\alpha(z)$, $\beta_2(z)$, $\gamma(z)$, $\beta_3(z)$, $\gamma_{\mathrm{r}}(z)$ and $\gamma_{\mathrm{s}}(z)$ represent the loss, GVD, self-phase modulation (SPM), third-order dispersion, stimulated Raman scattering and self-steepening parameters, respectively.

In this CV theory for DM solitons, we invoke some ideas developed in the CV treatments for localized modes in field theory and in condensed matter physics [10]. A fundamental point in CV theory is that one cannot simply substitute $\psi = f + q$, from equation (1), into the generalized NLSE (2), since the introduction of the CVs in f (as dynamical variables) will induce extra degrees of freedom, which can enlarge the available phase space of the system. Simply substituting $f + q$ into equation (2) would therefore introduce new and undesirable solutions into the system. One should constrain the system of new variables (CVs and q) so that the system must remain in the same phase space as the original field equation (2). The constraints are obtained by configuring the ansatz function f to being the best fit to the field ψ. In other words, CVs must be obtained by configuring that the ansatz function $f(X_1, X_2, \ldots, X_N)$, minimizes the functional $\mathcal{E}$, where

$$\mathcal{E} \equiv \int_{-\infty}^{\infty} |q|^2\,\mathrm{d}t = \int_{-\infty}^{\infty} |\psi - f(X_1, X_2, \ldots, X_N, t)|^2\,\mathrm{d}t. \tag{3}$$

In this CV approach, the quantity $\mathcal{E}$, which represents the *residual field* energy (RFE), serves as a measure for the correctness of the ansatz function f. Consequently, the RFE must be extremely small to give a proper physical meaning for the CVs. The constraints that we impose on the system will allow the CVs to evolve only in a particular direction to minimize the RFE during the dynamics in the following simple way

$$C_j = \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}X_j} = \int_{-\infty}^{\infty} \mathbb{R}\left[q\frac{\partial f^*}{\partial X_j}\right]\mathrm{d}t = 0\ (\text{or} \approx 0) \tag{4}$$

where $\mathbb{R}$ stands for real part. Here, the weak equality indicates that the constraints C_j need not be exactly zero. Note that the initial values of the CVs, $X_j(z=0)$, must be properly chosen to satisfy the constraint conditions i.e., $C_j(X_1, X_2, \ldots, X_N, z=0) \approx 0$. Then, we define a second set of constraints

$$\frac{\mathrm{d}C_j}{\mathrm{d}z} = 0 \quad (\text{or} \approx 0) \tag{5}$$

which guarantees that the first set of constraints C_j will be satisfied for all z, if they are initially satisfied. Substitution of equation (1) into the generalized NLSE (2) yields directly the equation of motion for the *residual field* as

$$\begin{aligned} q_z + \mathrm{i}\frac{\beta_2(z)}{2}q_{tt} - \frac{\beta_3(z)}{6}q_{ttt} - \mathrm{i}\gamma(z)|f+q|^2 q \\ +\frac{\alpha(z)}{2}q + \mathrm{i}\gamma_{\mathrm{r}}(z)(|f+q|^2)_t q + \gamma_{\mathrm{s}}(z)(|f+q|^2 q)_t \\ = -\sum_{j=1}^{N}\dot{X}_j f_{X_j} - \mathrm{i}\frac{\beta_2(z)}{2}f_{tt} + \frac{\beta_3(z)}{6}f_{ttt} + \mathrm{i}\gamma(z)|f+q|^2 f \\ -\frac{\alpha(z)}{2}f - \mathrm{i}\gamma_{\mathrm{r}}(z)(|f+q|^2)_t f - \gamma_{\mathrm{s}}(z)(|f+q|^2 f)_t \end{aligned} \tag{6}$$

where the overhead dot represents the derivative with respect to z and the subscript X_j denotes the partial derivative.

To obtain the CV equations of motion we follow Dirac's procedure [11]. According to this, a quantity which is weakly equal to zero (such as our second set of constraints) cannot be set to zero until all variations of the quantity with respect to the dynamical variables, to obtain the equations of motion have been performed. In other words, our second set of constraints must be applied only after obtaining the CV equations of motion. To this end, we write the second set of constraints as follows

$$\dot{C}_j = -\langle f^*_{X_j}\, q_z\rangle - \sum_{k=1}^{N}\dot{X}_k\langle f^*_{X_j X_k}\, q\rangle + \mathrm{c.c.} \tag{7}$$

Here, the bracketed term $\langle\cdots\rangle$ means $\int_{-\infty}^{\infty}(\cdots)\,\mathrm{d}t$. Next, we substitute the expression for q_z from equation (6), into the first term of the right-hand side of equation (7), to obtain the following matrix equation

$$[\dot{C}] = \left[\frac{\partial C}{\partial X}\right][\dot{X}] + [\mathcal{R}] \tag{8}$$

where

$$[X] \equiv \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} \qquad [C] \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix} \qquad [\mathcal{R}] \equiv \begin{bmatrix} \mathcal{R}_1 \\ \mathcal{R}_2 \\ \vdots \\ \mathcal{R}_N \end{bmatrix}$$

$$\left[\frac{\partial C}{\partial X}\right] \equiv \begin{bmatrix} \frac{\partial C_1}{\partial X_1} & \frac{\partial C_1}{\partial X_2} & \cdots & \frac{\partial C_1}{\partial X_N} \\ \frac{\partial C_2}{\partial X_1} & \frac{\partial C_2}{\partial X_2} & \cdots & \frac{\partial C_2}{\partial X_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial C_N}{\partial X_1} & \frac{\partial C_N}{\partial X_2} & \cdots & \frac{\partial C_N}{\partial X_N} \end{bmatrix} \tag{9}$$

with

$$\frac{\partial C_j}{\partial X_k} = 2\int_{-\infty}^{\infty}\mathbb{R}[f_{X_k} f^*_{X_j}]\,\mathrm{d}t - 2\int_{-\infty}^{\infty}\mathbb{R}[q\, f^*_{X_j X_k}]\,\mathrm{d}t \tag{10}$$

and

$$\mathcal{R}_k = \beta_2(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k} f_{tt}\rangle - \frac{\beta_3(z)}{3}\mathbb{R}\langle f^*_{X_k} f_{ttt}\rangle - 2\gamma(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k}|f+q|^2 f\rangle + \alpha(z)\,\mathbb{R}\langle f^*_{X_k} f\rangle$$
$$+2\gamma_{\mathrm{r}}(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k}(|f+q|^2)_t f\rangle + 2\gamma_{\mathrm{s}}(z)\mathbb{R}\langle f^*_{X_k}(|f+q|^2 f)_t\rangle + \beta_2(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k} q_{tt}\rangle$$
$$-\frac{\beta_3(z)}{3}\mathbb{R}\langle f^*_{X_k} q_{ttt}\rangle - 2\gamma(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k}|f+q|^2 q\rangle + \alpha(z)\,\mathbb{R}\langle f^*_{X_k} q\rangle$$
$$+2\gamma_{\mathrm{r}}(z)\mathbb{R}\langle \mathrm{i} f^*_{X_k}(|f+q|^2)_t q\rangle + 2\gamma_{\mathrm{s}}(z)\mathbb{R}\langle f^*_{X_k}(|f+q|^2 q)_t\rangle. \tag{11}$$

The matrix equation (8) corresponds to the CV equations of motion, in which the second constraint term appears explicitly in the left-hand side. At this stage, following Dirac's procedure [11], we set the constraint term to zero in equation (8) to finally obtain the following CV equations of motion

$$[\dot{X}] = -\left[\frac{\partial C}{\partial X}\right]^{-1}[\mathcal{R}]. \tag{12}$$

The set of equations (6) and (12) represent the complete CV treatment for the generalized NLSE (2).

On the other hand, it is interesting to note that the lowest-order approximation of the above CV theory, called 'bare approximation', is obtained by setting the *residual field* to zero [$q(z,t) = 0$]. In this case one can assume the desired form for the ansatz function f, such as a Gaussian profile given by

$$f = X_1 \exp\left[-\frac{(t-X_2)^2}{X_3^2} + \mathrm{i}\frac{X_4}{2}(t-X_2)^2 + \mathrm{i}X_5(t-X_2) + \mathrm{i}X_6\right] \tag{13}$$

where X_1, X_2, $\sqrt{2\ln 2}X_3$, $X_4/(2\pi)$, $X_5/(2\pi)$ and X_6 represent the pulse amplitude, temporal position, pulse width (FWHM), chirp, frequency and phase, respectively. Then, the equations of motion reduce to equation (12), with $q = 0$, and in which all the coefficients can be easily calculated using this Gaussian ansatz. Thus, we obtain the following explicit analytical expressions for the CV equations of motion:

$$\dot{X}_1 = -\tfrac{1}{2}\alpha(z)X_1 + \tfrac{1}{2}\beta_2(z)\,X_1X_4 - \tfrac{1}{2}\beta_3(z)X_1X_4X_5 \tag{14a}$$

$$\dot{X}_2 = -\beta_2(z)X_5 + \beta_3(z)\left(\frac{1}{2X_3^2} + \frac{X_5^2}{2} + \frac{X_3^2X_4^2}{8}\right) + \frac{3}{2\sqrt{2}}\gamma_{\mathrm{s}}(z)X_1^2 \tag{14b}$$

$$\dot{X}_3 = -\beta_2(z)\,X_3X_4 + \beta_3(z)X_3X_4X_5 \tag{14c}$$

$$\dot{X}_4 = -\beta_2(z)\left(\frac{4}{X_3^4} - X_4^2\right) - \frac{\sqrt{2}\gamma(z)X_1^2}{X_3^2} + \beta_3(z)\left(\frac{4X_5}{X_3^4} - X_4^2X_5\right) + \frac{\sqrt{2}\gamma_{\mathrm{s}}(z)X_1^2X_5}{X_3^2} \tag{14d}$$

$$\dot{X}_5 = \frac{\sqrt{2}\gamma_{\mathrm{r}}(z)X_1^2}{X_3^2} + \frac{\gamma_{\mathrm{s}}(z)X_1^2X_4}{\sqrt{2}} \tag{14e}$$

$$\dot{X}_6 = \beta_2(z)\left(\frac{1}{X_3^2} - \frac{X_5^2}{2}\right) + \frac{5\gamma(z)X_1^2}{4\sqrt{2}} + \beta_3(z)\left(\frac{X_5^3}{3} + \frac{X_3^2X_4^2X_5}{8} - \frac{X_5}{2X_3^2}\right) + \frac{\gamma_{\mathrm{s}}(z)X_1^2X_5}{4\sqrt{2}}. \tag{14f}$$

To illustrate this CV treatment for DM solitons, we demonstrate the pulse propagation in a typical DM fibre transmission line, with a periodic dispersion management using two types of fibre, as schematically represented in figure 1(*a*) (average GVD value is shown as a dashed line). GVD: 16.2 and −75 ps nm^{-1} km^{-1}, third-order dispersion: 0.057 and −0.175 ps nm^{-2} km^{-1}, losses: 0.2 and 0.44 dB km^{-1}, effective core areas: 80 and 17 μm^2, fibre lengths: 9 and 1.5 km and the nonlinear index coefficients for both type of fibres are taken as 2.7×10^{-20} m^2 W^{-1}. Figure 1(*b*) shows the evolution of $\mathcal{E}(z)/E(z)$ (i.e., the RFE $\mathcal{E}$ is normalized to the pulse

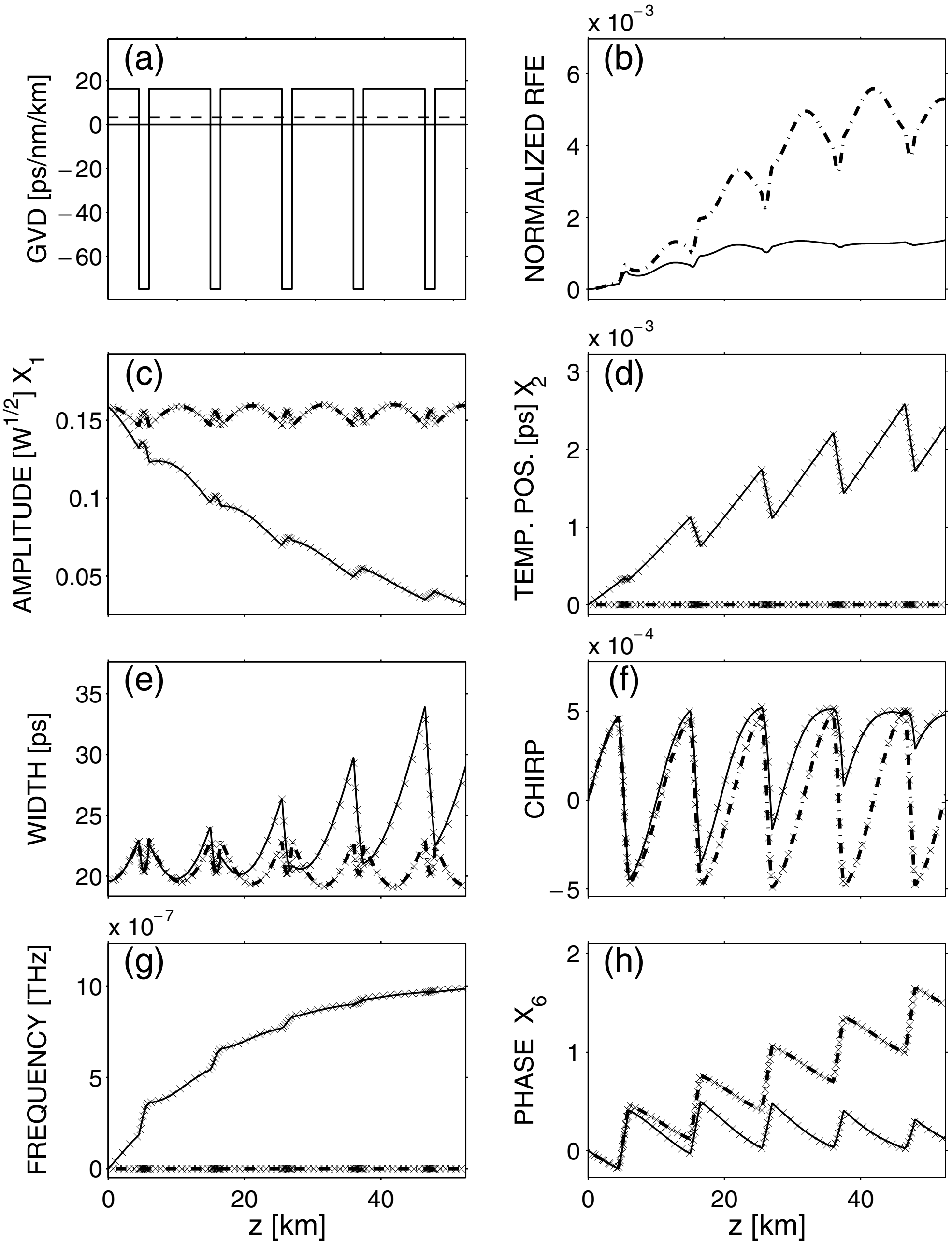

Figure 1. (*a*) Schematic diagram of the DM link using alternate positive/negative dispersion fibres. (*b*) Evolution of the normalized RFE $\mathcal{E}(z)/E(z)$ versus the propagation coordinate, z, for the generalized NLSE (solid curve), and for the basic NLSE (dot-dashed curve). (*c*)–(*h*) Evolution of the CVs X_j versus propagation coordinate z, for the DM fibre line considered in (*a*). Solid and dot-dashed curves correspond to generalized and basic NLSEs, respectively. The '×' marks represent the solution of equations (14) for both generalized and basic NLSEs. Pulse width, frequency and chirp are calculated by the formulae $\sqrt{2\ln 2}X_3$, $X_5/(2\pi)$ and $X_4/(2\pi)$, respectively.

energy $E = \langle |f+q|^2\rangle$), for five map lengths (52.5 km $\approx$ amplification length), when an initial Gaussian profile pulse, whose parameters are derived from the stationary solution (fixed point), corresponding to the basic NLSE (without linear and nonlinear higher-order terms), is launched at the mid-point of an anomalous-dispersion fibre. The dot-dashed curve shows the result obtained from the basic NLSE. The solid curve shows the result obtained from the generalized NLSE (2). In both cases, we observe that the RFE remains within the order of 10^{-3}, thus indicating that the Gaussian ansatz provides an approximate but fairly accurate representation of this DM soliton. Furthermore, during the dynamics, the maximum values for all the constraints C_j ($j = 1, 2, \ldots, 6$) did not exceed the value of 2×10^{-2} (we find a good validity of the constraint conditions as long as the maximum values of the constraints are below 10^{-1} throughout the dynamics). Thus, all the control parameters that serve as a measure for the correctness of this CV theory (RFE and constraints) are satisfied very well in this demonstration, which therefore gives proper physical meaning to the corresponding CVs, represented in figure 1(*c*)–(*h*). An interesting point in figure 1(*c*)–(*h*) is that the *bare approximation* obtained by directly solving the set of equations (14), represented by the '×' marks, agrees surprisingly well with the CV theory for the DM soliton under consideration. In fact, one might have expected such an agreement in view of the small magnitude of RFE as seen in figure 1(*b*).

Nevertheless, we would now like to emphasize the following fundamental point: bare approximation based on the Gaussian ansatz agrees very well with our CV theory for single-pulse dynamics, whereas this approximation may in contrast lead to very poor results in important practical problems, such as the modelling of soliton interactions. For example, in a recent study bare approximation led to a discrepancy of a factor of 2, with respect to a full numerical approach, in predicting the interaction of DM solitons [12]. Here, we discuss the problem of introducing the CVs in the coupled NLSE in view of modelling the interaction of adjacent pulses in the same channel of a transmission system. To study the interaction of two adjacent pulses, we consider the coupled NLSE of the form [13]

$$\frac{\partial\psi_l}{\partial z} + \mathrm{i}\frac{\beta_2}{2}\frac{\partial^2\psi_l}{\partial t^2} - \mathrm{i}\gamma|\psi_l|^2\psi_l + \frac{\alpha}{2}\psi_l = \mathrm{i}\gamma(2|\psi_l|^2\psi_{3-l} + \psi_l^2\psi_{3-l}^*) \tag{15}$$

where $l (= 1, 2)$ denotes each pulse. We decompose the fields (ψ_l) in the same way as in equation (1) but with the following ansatz

$$f_\pm(t,z) = X_1 \exp\left[-\frac{(t\pm X_2)^2}{X_3^2} + \mathrm{i}\frac{X_4}{2}(t\pm X_2)^2 \mp \mathrm{i}X_5(t\pm X_2) + \mathrm{i}X_6\right]. \tag{16}$$

Then, from the bare approximation, we derive the CV equations as

$$\dot{X}_1 = \frac{1}{2}(\beta_2 X_1 X_4 - \alpha X_1) - \frac{\gamma X_1^2 E}{8\sqrt{2}}[(10 - 4X_2^2/X_3^2 + X_3^2X_2^2X_4^2 - 2X_3^2X_2X_4X_5 + X_3^2X_5^2)s + 4X_2(X_2X_4 - X_5)c] \tag{17a}$$

$$\dot{X}_2 = -\beta_2 X_5 + \frac{\gamma X_1^2 E}{2\sqrt{2}}[X_3^2(X_2X_4 - X_5)c + 2X_2 s] \tag{17b}$$

$$\dot{X}_3 = -\beta_2 X_3 X_4 + \frac{\gamma X_1^2 E}{4\sqrt{2}}[4X_2X_3(X_2X_4 - X_5)c + (X_3^3X_2^2X_4^2 - 2X_3^3X_2X_4X_5 - 4X_2^2/X_3 + 2X_3 + X_3^3X_5^2)s] \tag{17c}$$

$$\dot{X}_4 = -\beta_2\left(\frac{4}{X_3^4} - X_4^2\right) - \frac{\sqrt{2}\gamma X_1^2}{X_3^2} + \frac{\gamma X_1^2 E}{3\sqrt{2}}\left[9\left(2X_2X_4X_5 - X_2^2X_4^2 + \frac{4X_2^2}{X_3^4} - \frac{2}{X_3^2} - X_5^2\right)c + \frac{X_2}{X_3^2}\left(X_2X_4 - X_5\right)s\right] \tag{17d}$$

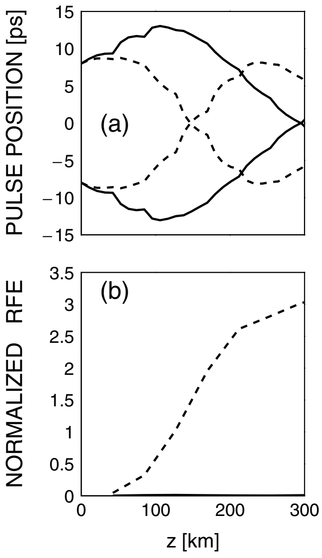

Figure 2. Plots showing the pulse interaction. (*a*) Evolution of the pulse positions versus propagation coordinate z. (*b*) Evolution of the normalized RFE $\mathcal{E}(z)/E(z)$ versus propagation coordinate z. Solid and dashed curves correspond to complete CV theory and the bare calculation, respectively.

$$\dot{X}_5 = \frac{\gamma X_1^2 E}{2\sqrt{2}}\left[\left(X_4 X_3^2 X_5 - \frac{12 X_2}{X_3^2} - X_4^2 X_3^2 X_2\right) c + 2\,(3X_5 - 2X_2X_4)\, s\right] \tag{17e}$$

$$\dot{X}_6 = -\beta_2\left(\frac{X_5^2}{2} - \frac{1}{X_3^2}\right) + \frac{5\gamma X_1^2}{4\sqrt{2}} + \frac{\gamma X_1^2 E}{8\sqrt{2}}[4X_2(5X_5 - 3X_2X_4)s + (30 - 10X_3^2X_2X_4X_5 + 7X_3^2X_5^2 - 12X_2^2/X_3^2 + 3X_3^2X_2^2X_4^2)c] \tag{17f}$$

where $s = \sin[(X_2X_4 - X_5)X_2]$, $c = \cos[(X_2X_4 - X_5)X_2]$ and $E = \exp[-1/4(X_3^4X_2^2X_4^2 - 2X_3^4X_2X_4X_5 + X_3^4X_5^2 + 12X_2^2)/X_3^2]$.

Figure 2 represents the dynamics of two pulses with the arbitrary initial conditions $[X_1 = 0.23,\ X_2 = \pm 8,\ X_3 = 9.88,\ X_4 = -0.0079,\ X_5 = 0,\ X_6 = 0]$. Solid and dashed curves represent the results obtained from our CV theory and bare calculation (17), respectively. Figure 2 demonstrates the general feature that the bare calculation based on the Gaussian ansatz (dashed curves) leads to wrong results for the modelling of soliton interactions than for single-soliton dynamics for an initial pulse separation of 16 ps. For example, we observe in figure 2(*a*) the following prediction for the collision distance: $Z_c = 294.7$ km (given by our CV theory). The bare calculation ($Z_c = 146.5$ km) leads to a huge discrepancy, of nearly 50%. The large discrepancy is because of the approximate nature of the bare calculation with the Gaussian ansatz in equations (17). Figure 2(*b*) show the spatial evolution of the normalized RFE for both complete CV theory and bare calculation. In both cases, the RFE execute an oscillating behaviour with an amplitude that increases continually as it approaches the collision point (for clarity we have plotted only the envelope of the oscillating RFE). More importantly, as one could have expected in view of the results in figure 2(*a*), the bare calculation leads to dramatically large values of the RFE (see the dashed curve in figure 2(*b*)). One can clearly observe in figure 2(*b*) a peak value that attains nearly 300% of the total energy of the two pulses, in the range $0 \leqslant z \leqslant Z_c$. The occurrence of these huge values of the RFE gives no physical meaning to the CVs obtained via bare calculation. Thus, it comes out from the results in figure 2 that the bare calculation with the Gaussian ansatz can lead to completely wrong results in some regions of the parameter space of the DM solitons. In such a situation, one must '*dress*' the Gaussian ansatz (by minimizing the residual field) to obtain an exact description of the solitons in optical fibres. In figure 2(*b*) CV theory leads to a RFE which does not exceed 1% of the total energy of the two pulses and therefore gives a correct physical meaning to the corresponding CVs.

In conclusion, we have demonstrated a method for incorporating the *residual field* in the CV treatment of DM solitons. This *residual field* allows one to account not only for the soliton dressing and any radiation coupled to the soliton's motion, but also for the insufficiencies in the choice of the soliton ansatz function. Incorporating the residual field in the CV treatment of DM solitons should ensure progress in the direction of a less phenomenological theory than those reported in the literature to date.

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